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Contribution of Khwârazmî to Mathematics and Geography

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CONTRIBUTION OF KHWÂRAZMÎ TO MATHEMATICS AND GEOGRAPHY*

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Islam gave a new civilization to the Turks who carried that from its centres into China in the east, India in the south east, Russia in the north, and Anatolia in the west of Asia. The Turks who had also transmitted that civilization to Europe and Africa had themselves been deeply affected by their conversion. Islamic ideology had penetrated the social, political, juridical and educational concepts of the Turks of Medieval period and their culture was endowed by the temporal and the cultural outlook of Islam. The Turks were always great admirers of learning but as the learning in the Middle Ages was synonymous for theology, the Islamic theology soon captivated their attention. Great madrasas which correspond to our modern universities were also founded by the Turks where scholars and students were to study which was confined to Arabic rhetoric, logic, philosophy, Islamic history, hadith, fiqh, tafsir of the Qur'an as well as mathematics. The language of the instruction of those institutions was Arabic; therefore generally literature on all these branches of knowledge was produced in that language.

The Turks were also great admirers of other sciences and have distinguished themselves in nearly all branches of knowledge. They have a definite position in the history of science and their contributions are vast and their effects are also far-reaching. Even the Uygur Turks (740-1335 A.D.) of Turkistan who were advanced and enjoyed a high level of cultures and civilization had shown great interest in scientific study. It is said that an Uygur work deals with movements of the stars in relation to the sun.[1] Another Uygur work on Cosmography describes the revolutions of the stars.[2] But one of the greatest Turkish minds of the medieval Islamic age is Abu Abdullah Muhammad bin Mûsâ al-Khwârazmî (b. before 800, d. after 847) who was a mathematician, astronomer as well as a geographer and a historian. It is said that he is the author of the oldest astronomical tables, the oldest work on arithmetic and the oldest work on algebra which were translated into Latin and were used until the sixteenth century as the principal mathematical text books in European universities. Originally he belonged to Khwârazm (modern Khiwa) situated in Turkistan but all his works are in Arabic language. Therefore, he is Turk in nationality but Arab in language. He was summoned to Baghdad by Abbasid Caliph Al-Ma'mun (213-833) who was himself a philosopher, a theologian and a great patron of learning. He had established his famous Bayt al-Hikma (House of Wisdom) which worked like a modern research academy. It had a large and rich library (Khizâna Kutub al-Hikma) and distinguished scholars of various faiths were assembled to produce scientific masterpieces as well as to translate faithfully nearly all the great and important ancient works of Greek, Sanskrit, Pahlawi and of other languages into Arabic. Mûsâ al-Khwârazmî, according to Ibn al-Nadîm[3] and Ibn al-Qiftî[4] as is quoted by Prof.

* Note: All images in the paper were newly introduced by the editor and are not part of the original paper.
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[1] *Turkish Architecture,* translated by Prof. Dr. Ahmet Edip Uysal Ankara 1965, p. 2.
[2] *Ibid.*
[3] *Fihrist al-Ulûm,* edited by Flügel, volume 1, 1871, p. 274.

Dr. Aydin Sayili,[5] was attached to (or devoted himself entirely to) Khizâna al-Hikma. It is also said that he was appointed court astronomer of Caliph Al-Ma'mun who also commissioned him to prepare abstracts from one of the Indian books entitled Surya Siddhanta which was called al-Sindhind[6] in Arabic.[7] Mûsâ al-Khwârazmî had also translated certain Greek works[8] into Arabic and produced his own scholarly works not only on astronomy and mathematics but also on geography and history. It was for Caliph al-Ma'mun that Al-Khwârazmî composed his astronomical treatise and dedicated his book on Algebra to that caliph.

Contribution to Mathematics

Mûsâ Al-Khwârazmî is one of the greatest scientific minds of the medieval period and most important Muslim mathematician who was justly called the 'father of algebra'. He wrote the Kitâb al-Jem wa'l Tafrîq bi Hisâb al-Hind also called Kitâb Hisâb al-adad al-Hindî on arithmetic in which he used Indian numerals[9] including zero in place of depicting numbers by the letters of the alphabet and the decimal notations or numeration by position" for the first time. It deals with the four basic operations of addition, subtraction, multiplication and division as well as with both common and sexagesimal fractions and the extraction of the square root. The original Arabic text of the book is lost and its only Latin translation is available.

Figure 1. *Mûsâ Al-Khwârazmî. The drawing of Khwârizmî on the stam. The stamp reads: Post USSR 1983, 1200 Years, Mukhammad al-Korezmi.*

[4] *Tarikh al-Hukama,* edited by Lippert, Berlin 1903 (Cairo edition, 1326 H), p. 286.
[5] Aydin Sayili, *The Observatory in Islam,* Ankara 1960, p. 55.
[6] Abdulhak Adnan Adivar, "Harizmi" in *Islam Ansiklopedisi,* volume 4, p. 261.
[7] It was first introduced in Baghdad by an Indian traveller in 771 A.D. which by order of al-Mansur was translated into Arabic by Muhammad ibn Ibrahim al-Fazârî between 796 and 806 for the first time.
[8] Stephen and Nandy Ronart, *Concise Encyclopaedia of Arabic Civilization,* New York I960, p. 295.
[9] Ya'qub bin Tariq was the first Muslim to introduce Indian numbers to Arabs.

Other mathematical writings of Mûsâ Al-Khwârazmî are also not known. His best known classical work on algebra is the Kitâb al-Mukhtasar fî Hisâb al-Jabr wa'l Muqâbala. It was also translated into Latin in the Middle Ages and holds an eminent place in the history of mathematics, firstly, in the words of Galal S.A. Shawki,[10] because it defined algebra as an independent disciple in mathematics, and secondly because it accelerated the introduction of the Arabic place value numbering into the West. The book is devoted to finding solutions to practical problems which Muslims encountered in daily life[11] concerning matters of inheritance, legacies, partition, lawsuits and commerce, with over eight hundred examples. The original work in Arabic was written in 820 A.D.[12] and was translated into Latin in the twelfth century. It is worth remarking that the term al-jabr, in the Latinized form of algebra, has found its way into the language of Europe, whiles the old mathematical term, algorism, is a distortion of al-Khwârazmî's name.

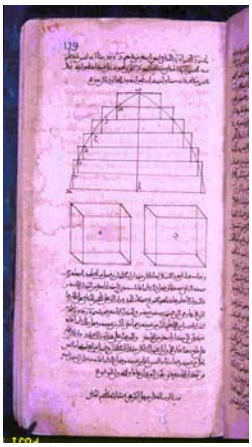

Figure 2. *A sample page of Suhayl al-Kûhî's manuscript Risâla-i Abi Sahl or Fî istihraci masaha al-muhassama al-mkâfî. Suleymaniye Library, Ayasofya 4832.*

The meaning of the Arabic word Al-Jabr is restoration by transposing negative quantities to the other side of the equation to make them positive and the term Al-Muqâbalah refers to the process of eliminating identical quantities from the two sides of the equation.[13] But the best translation for Hisâb al-Jabr wa'l-Muqâbala, according to John K. Baumgart,[14] is 'the science of equations'. The algebra of Mûsâ al-Khwârazmî was rhetorical in form. Al-Khwârazmî had given the rules for the solution of quadratic equations which are supported in a number of cases by geometrical proofs also. The unknown quantity, in the words of Galal S.A. Shawki, was termed the "thing" (shay) or "root" (jazr); the latter means in Arabic the origin or base, also the root of a tree, hence, the use of the expression "root of an equation" is derived from the Arabic concept.[15] Mûsâ al-Khwârazmî had used the Arabic word for root to denote the first degree term of a quadratic equation. Explaining in detail he says, "The following' is an example of squares equal to roots, a square is equal to 5 roots. The root of the square then is 5, and 25 forms its square, which of course

[10] Galal S.A. Shawki, *Formulation and Development of Algebra by Muslim Scholars,* published in *Islamic Studies* of Islamabad, volume 23, No. 4, p. 338.
[11] Lancelot Hogbin, *Mathematics for the Million,* New York 1946, p. 291.
[12] Sidney G. Hocker and others, *Fundamental Concepts of Arithmetic,* 1963, p. 9.
[13] Galal S.A. Shawki, Formulation and Development of Algebra by Muslim Scholars in Islamic Studies of Islamabad, volume 22, No. 4, p. 338.
[14] Historical topics for the Mathematics Classroom, Washington 1969, pp. 233-4.

equals 5 of its roots."[16] For the second power of a quantity he employs mâl (wealth, property) which is also used to mean only "quantity" and dirham is used as unit of coinage.

Figure 3. *A mathematical figure from Ali Kuscu's book Risâla al-Fathiyya fi ilmi'l-Hay'a. Süleymaniye Library, Ayasofya 2733.*

The bases of algebra, first viewed as an independent mathematical discipline, were laid down by Mûsâ al- illustrated his method of solution by practical examples.[17] He was quite aware of the existence of two roots of the quadratic equation, though he cared for positive, real roots only.[18]

His Hisâb al-Jabr wa'l-Muqâbalah is actually on applied mathematics. Its first part discusses the equations of the first and second degrees. All his proposed problems can be reduced to one of the six standard forms. He gives rules for the solution of each of the six forms and explains how to reduce any given problem to one of these standard forms with examples.[19] The second part of the book deals with practical mensuration by giving rules for finding the area of various plane figures including the circles, and for finding the volume of a number of solids including cones and pyramids. The third part concerns legacies as well as inheritance and is the longest. It consists entirely of solved problems which arise out of legacies.

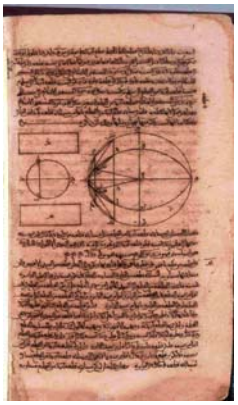

Figure 4. *A sample page of Sabit b. Kurra's manuscript Kitâb fuzû'al ustuwana wa natstabiha. Sulaymaniye Library, Ayasofya 4832.*

[15] Islamic Studies of Islamabad, volume 23, No. 4, p. 339.
[16] Philip S. Jones, "Large Roman Numerals", The Mathematics Teacher, volume 28, p. 261.
[17] Islamic Studies of Islamabad, volume 23, No. 4, p. 351.
[18] Ibid.
[19] G.J. Toomer, "Al-Khwârazmî", Dictionary of Scientific Biography, volume 7, p. 359.

The mathematical works of Mûsâ al-Khwârazmî were the chief text books used in European universities up to the seventeenth century. He is, in the words of ᶜAli 'Abdullah Al-Daffa,[20] the founder of algebra and had transformed the concept of a number from its earlier arithmetic character as a fixed quantity into that, of variable element in an equation. He also found a method to solve general equations of the first and second degree in one unknown by both algebraic and geometric means.[21] It was through his work on mathematics that the Indian system of numeration was known to the Arabs and later through its Latin translation to the people of Europe. **He synchronized Greek and Indian mathematical knowledge but was the first mathematician to distinguish clearly between algebra and geometry and gave geometrical solutions of linear and quadratic equations**.

Figure 5. *A sample page of Sabit b. Kurra's manuscript Kitâb fuzû'al ustuwana wa natstabiha. Suleymaniye Library, Ayasofya 4832.*

Contribution to Geography

Mûsâ al-Khwârazmî had also contributed to the science of geography. As the book of Geography of Claudius Ptolemy (2nd century A.D.) was translated several times into Arabic he had a model for writing his book in this field of knowledge. His book on geography entitled Kitâb Sûrat al-Ard (Book of the image of the earth) consists almost entirely of lists of longitudes as well as latitudes of localities and gives in a tabulated form the coordinates of the places like cities, mountains, seas, rivers, islands etc. The book is arranged according to the Greek system of the seven climes (aqâlim) giving contemporary data but the knowledge acquired by the other Muslims are also incorporated into it. The first section lists cities, the second, mountains (giving the coordinates of their extreme points and their orientation); the third, seas (giving the coordinates of salient point on their coastlines and a rough description of their outlines); the fourth, islands (giving the coordinates of their centres, and their length and breadth); the fifth, the central points of various

[20] The Muslim Contribution to Mathematics, London 1977, p. 7.
[21] Franklin W. Kokomoor, Mathematics in Human Affairs, New York 1946, p. 172.

geographical regions; the sixth, rivers (giving their salient points and towns on them).22 This book had served as a basis for later works and stimulated geographical studies and the composition of original treatises. It is said that his Kitâb Sûrat al-Ard was also accompanied by regional maps of each of the climes and by a single world map called "al-Sûrat al-Ma'muniyya" but have been lost. It is also said that his map of the world was the first map of the heavens and the world drawn by Muslims. But the editor of the Kitâb Sûrat al-Ard Hans von Mzik, has produced only four maps. These four maps, in the words of S. Maqbul Ahmad,23 seem to be later recessions of the original maps. But Ibrahim Shawkat24 reasons that since Al-Khwârazmî wrote a brief work on geography, he did not draw a complete map of the world but confined himself to draw only the four maps as an illustration. His source of inspiration might possibly have been the mappa mundi25 constructed for Caliph Al-Ma'mun by a team of geographers in which Al-Khwârazmî himself would have been included[26].

Figure 6. *The world map of al-Idrisi., Ahmad b. Sahl Al-BAlhî, Akâlim al-buldân, Suleymaniye Library, Ayasofya 2577.*

The Kitâb Sûrat al-Ard depends, even if in an indirect manner, on the Geography of Ptolemy,[27] but in the opinion of Ibrahim Shawkat, it was based on the work of Marinus.[28] The book was produced under the patronage of the Caliph Al-Ma'mun in about 830 A.D. in which the towns and mountains are presented in a tabulated form, and oceans, seas, islands, countries, springs and rivers are given in a descriptive form. Again, towns, mountains, springs and rivers are described according to the climes (aqâlim) to which they belong, while the description of the oceans and seas is free from the limits of these climes. Similarly islands are described under the seas and oceans to which they belong. The description of the countries is also free from the limits of the climes. Along with the geographical names of the Muslim period, a large number of ancient place names are also founded in the book but in the later portions these names rapidly begin to disappear.

[22] G.J. Toomer, "Al-Khwarizmi", Dictionary of Scientific Biography, volume 7, p. 361.
[23] Encyclopaedia of Islam, (new edition), volume 4.
[24] Khara'it Jughrafiqyyi al-'Arab al-awwel, Majallet al-Ustadh of Baghdad, 1962, pp. 7-8.
[25] Mappa Mundi is a term used for the map of the world.
[26] Encyclopaedia of Islam, volume 4, p. 1078.
[27] Ibid.

The map of the world of Mûsâ al-Khwârazmî called al-Sûrat al-Ma'muniyya has now been fully reconstructed by an Indian scholar, Dr. S. Razia Jafri,[29] on the basis of description and data given in his Kitâb Sûrat al-Ard. It is divided into 38 sections which are again sub-divided into 1740 small squares from West to East and into 1200 small squares "from South to North. Each clime (iqlîm) from West to East is again divided into seven sections. It is to be noted that the general division of the Map into climes is according to al-Khwârazmî, but the sub-division of the climes into section is done by Dr. Razia Jafri arbitrarily. In this way it is just like an Atlas. It is to be noted that the Soviet Academy of Sciences of Tajik is publishing it along with the forward and introduction of Dr. Kamal Ayni and Prof. S. Maqbul Ahmad respectively. The printing of this work is done under the supervision of Prof. M.S. Asimov who is the eminent scholar and the president of the Academy of Sciences of the Tajik SSR at Dushanbe.

Mûsâ al-Khwârazmî is the author of several other books on astronomy and history. He became well known as a mathematician and it is said that he is the author of the oldest work on algebra. But the Professor of the History of Science, Dr. Aydin Sayili says in one of his research papers entitled "Turkish contribution to Science" as follows: "Abu'l Fadl ᶜAbdulhamîd ibn Wâsiᶜ ibn Turk was apparently the first Islamic mathematician to write a book on algebra. Indeed, he, very likely, wrote his algebra before Al-Khwârazmî wrote his. For unlike Al-Khwârazmî, he did not write an unabridged algebra, and, moreover, there is evidence that Al-Khwârazmî was still alive at about the middle of ninth century. ᶜAbdulhamîd ibn Turk was also the author of certain books on numbers, on commercial arithmetic, and on the art of calculation, probably with the decimal system.[30] Now, it is not possible for me to agree or disagree with him but it is realistic to say that the works of Mûsâ al-Khwârazmî on mathematics have great influence in the birth of Western Science and he is rightly called the "father of algebra" and a peerless geographer.

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[28] Tafkir al-ᶜArab al-Jughrafi wa'itaqat al-Yunan bihi", extract from the Journal "A l-Ustadh", Baghdad 1961.
[29] One of the staff members of Aligarh Muslim University of India.
[30] Aydin Sayili, "Turkish Contributions to Scientific Work in Islam", *Belleten* (Turkish Historical Society), volume 43, Ankara 1979, s. 16.

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